"*On Prognosis*" by William Farr (British Medical Almanack 1838; Supplement 199–216) Part 2 (pages 208–216)

Typed and edited by Gerry Bernard Hill

Introductory Note by Alfredo Morabia

Part 1 of "*On Prognosis*" appeared in Soz Praventiv Med 2003; 48(4): 219–224. To make this second part easier to read we have added table numbers in brackets and added the references to these tables in the text. The commentary by B. Burt Gerstman, appearing in this issue, is of great help to the modern reader as it gives the modern equivalent of the terms used by William Farr.

"On Prognosis" (continuation) by William Farr

The relative *force of mortality* is an unusual term; but it implies the rate of dying – the number deaths out of a given number, living a given time – and is required by the present state of science. The force of mortality in Sweden was 2482; this was the number of deaths out of 100 living a year – the assumed unity of time. Every population comprises two classes, the healthy and the sick; and it has been found that among adults there are two years of sick time to every death; in other words, that in a society where 1 dies annually 2 are constantly incapacitated by sickness. In Sweden, therefore, 4564 in 100 of the population constitute the sick, 95436 the healthy population. The deaths occur in the sick population; so the mean force of mortality may be taken at 0.50; of 100 living and sick 50 die annually. This is the average force of mortality in all kinds of sickness. Among the East India Company's labourers (= 20343 years of life) there were 328 years of sickness, and 973.5 years on the pension list; in the former 496, in the latter 161, deaths occurred.

Sick time 1391 years, deaths 657; annual deaths per cent 50.6.

The Liverpool Friendly Society[15]

Received sick pay 1 year 613; deaths 356; annual deaths per cent 58.1. On surgeons' books 1 yr. 912; deaths 408; annual deaths per cent 44.7.

[Table 1] Prussian Army.

	Years sickness	Ann. deaths per cent	Ann. invalided per cent
1821–30 (11 enumerations)	44810	24.47	10.2
1831–2 (3 enumerations)	18820	41.02	9.2

Assuming the mean strength of the Prussian army to be 100000, it follows that, in 1821–30, there were 0.015, in 1831–2 more than 0.18 constantly sick; that the absolute mortality was 1.23 in the first, 3.5 in the second period [Table 1]. The annual mortality among men, of every shade of sickness, was only 27.47 per cent.

In determining the relative force of mortality in different diseases, the same method is to be observed; the deaths must be divided by the sick time. As an illustration, it will be seen in the annexed table [Table 2] that 990 deaths occurred in a mean constant population of 41 individuals suffering from cholera. In a Cholera Hospital, containing 100 patients, on an average 2415 deaths would occur in the year. The second column, intended to represent the mean sickness from each particular disease, is the sum of the numbers *remaining* on December 31, 1826, 1827, 1828, 1829. To have made the numbers accurate, monthly enumerations should have been obtained of the numbers sick from each disease.

The table [Table 2] is deduced from a return given by Mr. H. Marshall, in the "Edinburgh Medical and Surgical Journal"[16], of the sickness among 69850 native troops in the Madras army. It must be considered rather an illustration than an accurate

[15] The total deaths were 408, but 26 occurred in the half year after admission before the members could receive sick pay, which they were only entitled to in two months; as a correction, 52 deaths were deducted.

[16] Vol. XXXIX., pp. 124, 135.

Soz.- Präventivmed. 48 (2003) 279–284
0303-8408/03/050279–06
DOI 10.1007/s00038-003-3005-7

[Table 2] Table of Sickness among the Native Troops of the Madras Army.

	Cases [a]	Sick a year [b]	Die [c]	Deaths in 100 cases [d]	Deaths in 100 sick a year [e]	Mean duration of cases in days [f]
Cholera	2142	41	990	46.2	2415	7
Apoplexy	76	3	51	67.0	1700	14
Dysentery	2285	136	198	8.7	146	22
Fever remittent	5067	185	244	4.8	132	13
continued	2627	71	73	2.8	101	10
intermittent	36436	1658	426	1.2	26	17
External inflammations	10803	381	52	0.5	17	13
Rheumatism	16387	1624	239	1.6	15	36
Ulcers	16482	1918	203	1.2	11	43
Syphilis	8668	970	55	0.6	6	41
All diseases						
Native troops	157796	9998	4041	2.6	40	23
English troops	84713	4371	2274	2.7	52	19

d = c:a; e = (c:b) × 100

view of the facts. The relative force of mortality did not differ widely from that prevailing among the sick European troops, although the annual mortality was 0.048 of the strength among the latter, 0.014 among the former; and the constantly sick among the native troops was 0.0334; among the English troops, 0.0924. The last column but one shows the relative force of mortality in different diseases; cholera and apoplexy are at the top, syphilis is at the bottom of the scale. The *mortality* and the *force of mortality* will readily be distinguished, by comparing cholera with consumption; the *mortality* in the latter is 90–100 per cent, but its mean duration is two years, and the force of mortality is consequently nearly 0.50; the mortality in cholera is not 50 per cent, while the *force of mortality* is 2415, for cholera destroys in a week as many as phthisis consumes in a year. Phthisis is more dangerous than cholera; but cholera, probably, excites the greatest terror.

The form as well as the nature of diseases must be taken into account, in estimating the degree of danger. Thus, at the

[Table 3] Table of Sickness, showing, in Small Pox, the number of cases remaining at the end of every fifth day [C]: the numbers to recover [A] and to die [B]; also the numbers that recover [D], die [E], terminate in the next period.

Days 0	To Recover A	To Die B	To Recover or To Die C	Recover D	Die E	Terminate F
5	3488	1780	5268	2	164	166
10	3486	1616	5102	5	737	742
15	3481	879	4360	37	552	589
20	3444	327	3771	176	153	329
25	3277	174	3451	321	70	391
30	2956	104	3060	487	39	526
35	2469	65	2534	535	23	558
40	1934	42	1976	466	14	480
45	1468	28	1496	384	9	393
50	1084	19	1103	246	3	249
55	838	16	854	367	4	371
60	641	14	655			
65	471	12	483	179	2	181
70	379[17]	11	390			
75	292	10	302	292	10	302
80	224	9	233			
85	173	8	181			
90	132	7	139			
95	100	6	106			
100	79	5	84			
105	60	4	64			
	200	20	220	added to the columns		

[17] Interpolated by successively multiplying by 0.76; which applies as high up the column as 1084. I have calculated a theoretical table that corresponds with the numbers observed.

Small Pox Hospital, (1836) 81 of 205 confluent cases terminated fatally; of the 116 remaining cases of modified confluent, and distinct variola, and varicella, two died of erysipelas and croup. Of 13 cases, confluent, malignant, and petechial, 12 were fatal.[18]

d. Periods of Disease

The laws of death and recovery, in different stages of disease, form the most interesting part of this inquiry. A very simple tabular construction enables us to determine the nature of these laws, and the probability of recovery at any period. Thus, I took 5268 cases of small pox from the books of the Small Pox Hospital, and noted the deaths and recoveries (cols. D, E) taking place every five days from the date of admission [Table 3]. 3488 recovered, 1780 died. Of the 1780 fatal cases, 164 died in the first five days after admission, leaving 1616 cases to enter on the subsequent period; and thus, by subtracting the deaths every successive five days, (col. E) the number of fatal cases remaining at every stage was obtained (col. B) Column A was obtained in the same manner, by subtracting, successively, the number recovering every five days (col. D) The table is read thus: Of 5268 cases of small pox remaining at the end of the fifth day, 3486 will recover, 1780 will die; and two will recover, 164 will die, in the next five days, leaving 5102 on the list at the end of the tenth day.

The cases are from the London Small Pox Hospital (1780-99); the time at which they entered has been assumed to be the end of the fifth day, as this has latterly been the period of admission. The day of the disease, and the day of the eruption at the time of admission, have been entered in the books for several years, by Dr. Gregory, and this is the result [Table 4]:

[Table 4]

Date	Cases [A]	Day of Disease [B]	Day of Eruption [C]	Interval in Days [D]
1827–31	1000	6333	3358	2975

[B] = 5333 + [A]; [D] = [B] – [C]

The *day of the disease* is one day more than the *duration of the disease*, which was, therefore, 5333 days at the time of admission. Omitting the fractions, patients enter the hospital at the end of the fifth day; at which period the table commences. The patients were not discharged till they had perfectly recovered, for fear of infection.

The probability of recovery is shown by the numbers of column A and column B, in juxtaposition. At the end of the fifteenth day there were 4360 cases – 3481 to recover and 879 to die. The chance of recovery is 3481 to 879 (4 to 1); at the 30th day the probability of recovery has risen; it is 2966[19] to 104 (29 to 1). The fraction expressing the probability of recovery is 2956/3060; the probability of dying is 104/3060; both probabilities added together make certainty.

The *probability of dying constantly decreases* in acute diseases; as the deaths take place at an earlier period than the recoveries. So the danger of being shot is greater at the beginning than in the middle of a battle; for a man alive in the middle of the day has escaped all the dangers of the morning.

This table [Table 3] possesses many other curious properties. I can here only call attention to two. It shows how long the disease will *probably* last from any given day; for 5268 cases are reduced to less than half the number (2534) by the 35th day. On the 6th day (end of the fifth) it is therefore *probable* the disease will have terminated by the 35th. On the 30th day it may be said that the disease will probably terminate in 15 days; for 3060 cases remaining at the end of the 30th, are reduced to 1496, or less than half, at the end of the 45th day. The probability that the patient will *recover* in the 15 days is (2956 – 1468)/3060 = 1488/3060; the probability that he will die in the time is 76/3060.

The *mean future duration* of the cases that attain the 5th, 10th, 15th, and every successive day is determined by adding the columns A, B, or C up to the number against the day in question; dividing by that number, and subtracting 0.5 from the quotient. For example: the addition of col. C up to 3060 (this included) against the 30th day produces the sum 13880; which divided by 3060 is 4.433; reduced by subtracting 0.5 is 3.933 – or nearly 4 periods of 5 days. Multiply 3.933 by 5 and the product will be 19.7, the mean future duration of cases in days.

The mean future duration of (a) cases of recovery, (b) fatal cases, and (c) all cases of Small Pox; also (d) the probability of dying at the end of the 5th, 10th, 20th, 30th and 40th day is here given [Table 5].

[Table 5]

Day	Expected Duration of			Probability of Dying
	Recoveries a	Fatal Cases b	All Cases c	d
5	41.5	12.0	31.5	0.3379
10	36.5	7.9	27.4	0.3168
20	26.9	11.0	25.5	0.0867
30	20.3	15.8	19.7	0.0340
40	18.4	22.6	18.5	0.0213

d = B : C in [Table 3]

[18] Dr. Gregory, "British Annals of Medicine".

[19] (ed.) Apparently a misprint for 2956

[Table 6] Table of the deaths and recoveries out of 1000 patients – this number being constantly kept up – in fourteen stages of Small Pox.

Days	a Constantly Sick	b Die in 5 days	c Recover in 5 days	d Daily deaths in 1000 (observed)	e Daily deaths in 1000 (calculated)	f Daily Recoveries in 1000
5–10	5135 [5185]	164	2	6.39		0.08
10–15	4681 [4731]	737	5	31.18		0.21
15–20	4066	552	37	27.16	27.16	1.82
20–25	3611	153	167	8.48	8.48	9.25
25–30	3255	70	321	4.30	4.30	19.72
30–35	2797	39	487	2.79	2.78	34.82
35–40	2255	23	535	2.04	2.04	47.46
40–45	1736	14	466	1.61	1.58	53.68
45–50	1299	9	384	1.38	1.27	59.52
50–55	978	3	246	0.61	1.03	50.50
		10 days	10 days			
55–60	668	4	367	0.60	0.84	54.94
60–65					0.69	
65–70	392	2	179	0.51	0.57	45.66
70–75					0.47	

[Apparent misprints shown in brackets]

The force of mortality in different diseases has been examined (page 208 [p. 279 of this issue]) it differs in intensity in the course of the same disease, according to a law which I have discovered in Small Pox. To determine the force of mortality at different periods, refer to the preceding table (page 210 [Table 3]): it will be perceived that 5268 cases remained at the end of the 5th day, 5102 at the end of the 10th day; and that 164 died in the 5 intervening days. The mean number constantly sick [Table 6], was the mean of the numbers at the beginning (5268) and the end (5102) of the period = 5315; so that 164 deaths took place in 5 days out of 5135[20] constantly sick, between the 5–10 days of Small Pox. This is equivalent to 6.39 daily deaths in 1000. The same method was pursued in calculating all the numbers in col. *D*; and the rate of recovery col. *F*.

The rate of mortality increased from the 5–10 days to days 10–15 when it attained a maximum (31.18); it decreased in a determined progression from the next period (15–20 days) to the end. The decrease begins to take place in geometrical progression; but the tendency to decrease is met by another force that neutralises part of its effect. This is illustrated in the annexed diagram: where the lines against each day show the relative force of mortality – the quantity eliminated daily by death out of a given constant quantity (1000) sick on the 18th, 19th, &c. days of the disease. The curved line (a) describes the actual course of the rate of mortality, the dotted line (b) the course the rate of mortality would pursue if uninfluenced by a second force, tending to keep up the original intensity. This curve agreeing with the facts observed was calculated and any point being known the rest of the course can be deduced by a very simple process. (See Diagram next page)

The diagram was calculated from 4915 cases of Small Pox (age 10–35);[21] since then I have discovered a simpler formula applying from 10–15 to the end of the disease and its terminations. The results (col. E) all calculated from one basis, agree so exactly with the facts observed (col. D) as to leave little doubt that the force of mortality changes according to a fixed law (Table [6], page 212). The nature of this law will be best understood by going over the calculation.[22]

The mean rate of mortality in the first period (days 15–20) is 27.16; and 27.16 multiplied by the rate of decrease, 0.312, is 8.48; the rate of decrease now changes to 0.508. To find the successive rates of decrease another rate must be employed; a rate regulating these rates. This second rate may be called

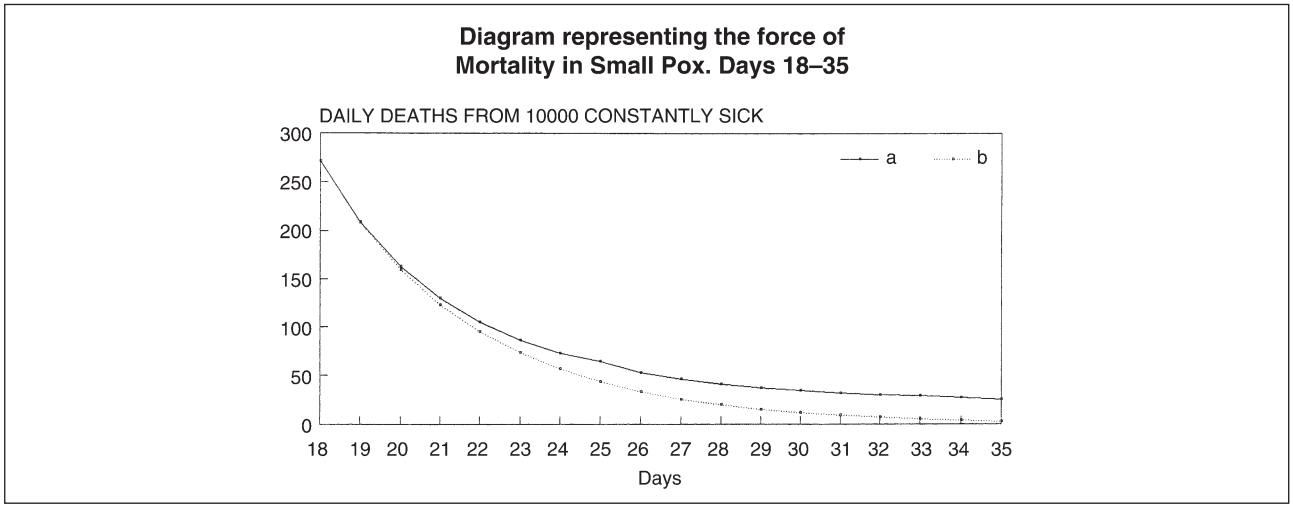
[20] (ed.) Apparently a misprint, (5268 + 5102) : 2 = 5185.

[21] British Annals of Medicine, page 74.

[22] The series of numbers in Table [6] p. 212 may be calculated in many ways; the following is one by logarithms [λ].

Rate of Mortality λ		Rates of Decrease λ	Constant λ
27.16	1.43392		
	–1.49429	–1.49429	
8.48	0.92821	0.2112	0.2112
	–1.70549	–1.70549	
4.30	0.63370	0.1056	0.1056
	–1.81109	–1.81109	
2.78	0.44479	0.0528	0.0528
	–1.86389	–1.86389	
2.04	0.30868		

By dividing the constant by 2, this is continued. In ascending, the operation is reversed.



the constant; it is here 1.626; the square root of 1.626 is 1.275, the next constant; and each successive one is the square root of the constant preceding. After deducing the roots of the constant, the successive rates are formed by them from the first; for 0.312 multiplied by the first constant 1.626, becomes 0.508; and 0.508 multiplied by 1.275, the square root of 1.626, becomes 0.647; so the successive rates of decrease are formed to the end of the series. The rates of mortality are determined by multiplying them by the rates of decrease; the rate of mortality at 20–25 days is 8.48; which multiplied by 0.508, produces the mortality of the next period, 4.30; and 4.30 multiplied by 0.647 = 2.78 the mortality of the next period. The constant – some power – or some root of it – regulates the whole series. The tendency to recover increases according to a determined law.

The Paris Board of Health gave the time at which 4907 fatal cases of cholera terminated as in col. B [Table 7]; we have added col. A, the number remaining at the end of every period, that it may be compared with the corresponding column in the Table of Small Pox.

[Table 7] Table of 4907 fatal cases of Cholera, showing the number remaining at each of 16 periods; and the number dying in the period following.

Hour	A To die	B Dying[23]		Day	A To die	B Dying
0	4907	204	2384*	5	576	125
6	4703	615		6	451	79
12	4088	392		7	372	171
18	3696	1173		8	201	35
Day	A	B		9	166	36
1	2523	823		10	139	111
2	1700	502		15	19	19
3	1198	382		20	0	
4	816	240				

[* Number of deaths from hours 0–18.]

The tendency to speak in weeks, and well known periods, produced the irregularity at the 7th day. Of the 171 thrown on that day, some died a day or two before, some a day or two afterwards. For the same reason it may be safely admitted that the deaths increased regularly on the first day. The daily rate of mortality in the first 12 hours was 16 per cent; in the next 12 hours (12–24) 37 per cent; in the second day 11 per cent; in the 3rd day 8 per cent, if the mortality of cases of cholera in Paris was 49 per cent – it could not have been higher – and none of the severe cases were cured in the first 3 days. The force of mortality attained its maximum in cholera by the 21st hour (18–24 hours); the maximum intensity in small pox is attained in days 10–15; in phthisis in 6–9 months. Taking a year as the unity of time, the relative maximum force of mortality – the deaths out of 100 constantly living – in the height of these three diseases is: cholera, 13614; small pox, 1150; phthisis, 118. The danger of cholera decreases as the time advances; the longer a cholera patient lives, the more likely he is to live. The way in which the prognosis becomes favourable is shown in the following table [Table 8].

[Table 8] Table of the Probability of Recovery from the severer Attacks of Cholera at the end of 12 hours, and 1, 2, and 3 days.

	Cases	To Recover	To Die	Probability of Recovery
0 hours	10000	5093	4907	0.509 nearly 1 to 1
12 …	9181	5093	4088	0.555 … 1.3 to 1
1 day	7616	5093	2523	0.669 … 2 to 1
2 …	6793	5093	1700	0.750 … 3 to 1
3 …	6291	5093	1198	0.809 … 4 to 1

[23] Rapport sur la Marche et les Effets de Cholera Morbus dans Paris, et le Département de la Seine.

These facts prove that in cholera the probability is generally not in favour of death; they also establish the importance of early treatment, for half the deaths happen in the first 24 hours. What the practitioner does he should do quickly.
The influence of age, sex, disease, different types of the same diseases, the period of disease in the prognosis have now been examined; the results are sufficiently decisive; they prove that these laws are active causes, to which the medical man should never forget to refer in practice.

Address for correspondence

Dr. Gerry B. Hill
263, Chelsea Road
Kingston, Ontario
Canada K7M 3Z3

Soz.- Präventivmed. 48 (2003) 279–284