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Reviews and Commentary

ON THE REDISCOVERY OF A DISTINCTION

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In 1980, Morgenstern et al. (1) described in detail how in epidemiology two different measures of disease incidence are used: risk and rate. For background, the authors cited mainly recent epidemiologic publications from the United States.

It is the purpose of this commentary to propose that the roots of the distinction between risk and rate stem from an earlier period; the renewed description can be qualified as a rediscovery of concepts which were well known to medical epidemiologists in the first half of the 19th century.

When reading some of the original papers by William Farr (1807–1883), one is surprised to find a clear and accurate description of the two different ways of measuring mortality (and morbidity) which Farr named the "probability of dying" and the "rate of mortality" (2, p. 490). Farr explained that the first has the nature of a probability and also named it a "death-chance." Of the second he stated, "The rate of mortality—or, in a technical sense, the mortality—expresses the ratio between three elements: 1) men living; 2) time; and 3) men dying. The men living and the *time expressed in years*, multiplied into each other, produce the years of life with which the deaths are compared. A *year of life* is the lifetime unit. It is represented by one person living through a year. Any number of persons living, one at a time, *in continuous succession* through a year, yield also one year of life. There are 525,949 minutes in a year; and 525,949 persons living through one minute also enjoy *one year* of life" (2, p. 485) (original italics). In one of his worked-out examples, Farr went on, "By the English Life Table 1,000 infants followed through their first year of age yield nearly 903 years of life; and the mortality is at the rate of 149/903, or, more correctly, 149493/902781 = 0.16559. It is 16.559 per cent per annum. The probability of dying is 0.149493; and upon the erroneous assumption that this is the rate of mortality it would be 14.949 per cent per annum; less by 1.610 than the true rate, *with which it should never be confounded*" (2, p. 491) (italics added).

Farr gives some interesting examples on mortality rates in different social classes, among the widowed and even among patients in hospitals, trying to calculate a prognostic index. He also gives some useful properties such as the formula for the calculation of the probability of dying from the rate of mortality and vice versa. Finally, he is aware that when the number of deaths

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is small, relative to the population studied, both measures approach each other numerically. As one of his main sources, Farr cites the works by Joshua Milne.

When reading Milne's 1837 treatise on "The Law of Mortality" (3), we find a complete nonmathematical account of the theory of the stationary population and of the fact that under such an assumption the mortality rate calculated from population statistics is the same as the one calculated from the follow-up of a cohort. Milne goes on to describe that the stationary state assumption is in practice easily tenable as long as increase or decrease of a population proceeds gradually and not "by sudden starts." Finally, Milne describes how one and the same technique can be used not only "amongst the whole of the inhabitants," but also to calculate age- and sex-specific mortality rates from any given cohort, such as subscribers to a life insurance. He gives a completely worked out example on the procedure (3, p. 550). Both Milne and Farr refer to French mathematicians and actuaries for the finer mathematical detail, notably Deparcieux and Demoivre. With only slight alterations, excerpts from the mentioned texts by Milne and Farr would pass largely unnoticed in any modern textbook of epidemiology, if it were not for their exceptionally clear use of the English language.

After this brief historical excursion, one is left wondering how this early 19th century knowledge got so lost that it needed repeat rediscovery (4, 5) by 20th century epidemiologists. Part of the blame for "this situation"—as Morgenstern et al. (1) call it in the introduction of their paper—might be with elementary introductions in biostatistics and epidemiology which might have been kept too elementary. When browsing through such introductions to statistics in medicine or through elementary textbooks of epidemiology, one usually finds that the rate of mortality is treated as a peculiar kind of probability or as if it were a case fatality rate, with very little awareness of the underlying theoretic assumptions. This contrasts with introductory textbooks on demography and actuarial science in which the 19th century distinctions are clearly preserved, unfortunately often buried under not so elementary calculus.

To an epidemiologist, the final question is how to prevent a relapse into ignorance. Let me propose, as a way of secondary prevention, that any student of biology, medicine, or biostatistics be made familiar with the basic concepts of the probability of dying and the rate of mortality, preferably in a nonmathematical language and with due reference to the pioneers of our trade. If excerpts from their writings are adapted, it need not take more than two or three pages of one of todays fine-printed textbooks of medicine.

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