

REITERATIONS

Death-rates in Great Britain and Sweden. Some general regularities and their significance*

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The object of this paper is to draw attention to certain regularities in the vital statistics of Great Britain, and to indicate their possible significance in interpreting the past and predicting the future. For purposes of comparison the vital statistics of Sweden have been examined from a similar point of view, and it will be shown that corresponding regularities seem to exist, though partly obscured by some influence the nature of which we are not in a position to identify.

The raw material of this investigation consisted of the specific death-rates at each period for England and Wales from 1845,¹ for Scotland from 1860,² and for Sweden from 1751,³ these being the earliest periods for which reliable statistics are available. As the effects which it is intended to investigate are the general tendencies, and as emphasis is not being laid on small variations, it has been considered sufficient to use where convenient for the whole population the mean of the figures for males and females respectively, whilst, when the statistics available refer to five-yearly periods of age—e.g. 5–10 and 10–15 years—the simple mean has again been taken as the figure for the 5–15 year period. As only the relative death-rates—that is the ratios—are important, as is shown below, any slight errors involved will in large part be automatically eliminated.

It is to be noted that the English figures are grouped in periods of ten calendar years, 1841–50, 1851–60, etc, and these have been centred for convenience at 1845, 1855, etc. The Swedish figures are similarly grouped in ten-yearly periods commencing with 1751–60 centred at 1755, but in this case the last figure (for 1925) is obtained by taking the average of the three years 1924, 1925, and 1926 as the figures relating to the whole decennium were not readily available. The most convenient Scottish figures² happened to be those obtained by taking the averages for three successive years—e.g. 1860, 1861, and 1862, centred at 1861—and there seemed to be no disadvantage in working in this one case with figures arranged on a slightly different basis.

Relative mortalities in Great Britain

In Table 1 are shown the specific death-rates per 1000 for each age-period of ten years for the earliest period regarding which

Table 1 Death-rates during the 'Standard Periods'

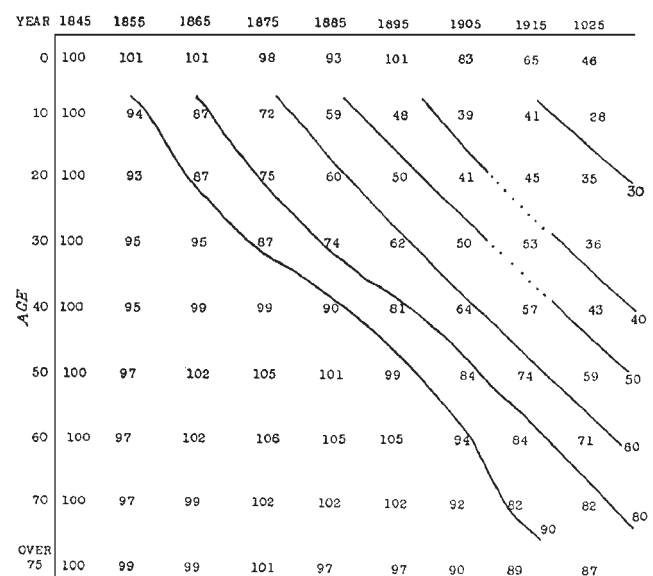
	Age-periods						
	5–15	15–25	25–35	35–45	45–55	55–65	65–75
England, 1841–50	7.1	8.4	10.3	12.9	17.0	29.9	63.6
Scotland, 1860–62							
Males	7.2	8.7	10.3	12.3	17.2	29.5	64.6
Females	7.0	7.6	9.6	11.5	14.4	24.9	56.5
Sweden, 1751–60	9.5	7.2	10.5	13.8	19.4	32.9	66.3

statistics are available to us. These we shall refer to as the standard periods.

In Tables 2, 3, and 4 are shown what we propose to call the relative mortalities for England and Wales, Scotland, and Sweden respectively. These relative mortalities have been obtained by calculating the corresponding specific mortality as a percentage of the specific mortality for the same age-group in the same country in the standard period.

We shall first of all consider the cases of England and Wales, and of Scotland. Superficial observation of Tables 2 and 3 shows at once that, with the exception of those relating to infants which will be discussed later, there is a general tendency for numbers of approximately the same magnitude to be arranged

Table 2 England and Wales: relative mortalities. (The figures in the zero row refer to deaths under one year per 1000 births.)



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Table 3 Scotland: relative mortalities

YEAR	1860	1870	1880	1890	1900	1910	1920	1930
0	100	104,5	97,5	104	102	88	75	64,5
10	100	108	85	69	55	45	35,5	30
20	100	104,5	97	76,5	63,5	47,5	42	34,5
30	100	109	91	86	79	56	50	39,5
40	100	111	98,5	100,5	91	69	59	51
50	100	114	108	115,5	110,5	88,5	73	66
60	100	110	110	126,5	125,5	103,5	91	81
70	100	97	96	110,5	108,5	97,5	90,5	88,5
OVER 75	100	96,5	88,5	98	94	80	90	91

diagonally in the Tables. In the earlier periods, especially in the case of the higher age-groups, the figures for England and Wales are all in the neighbourhood of 100. In the case of Scotland some of the earlier figures in the later age-groups rise somewhat above 100, the highest being 126.5. It is clear that the fall in mortality has begun at progressively later periods of time with increasing age.

In order to find out the nature of the regularities involved, contours have been sketched out in the figures corresponding to relative mortalities of 90, 80, 70, 60, ... per cent. It will be seen that these contours show a remarkable tendency to follow the diagonals. Furthermore, it will be noted that the deviations down the diagonals are in general not systematic errors in one direction or the other, but suggest rather the effect of random causes superimposed upon a constant mean. It is now to be noted that a diagonal line in the diagram represents the course of a group of people all born in a particular year. For instance, a person born in 1865 will be 10 years old in 1875, 20 in 1885,

30 in 1895, etc. Thus the figures along a diagonal represent the rates experienced by a particular group (or generation) of individuals all born in a particular year-period, as they pass through successive stages in their lives. The general conclusion then from the diagrams would seem to be that, apart from more or less random deviations, the relative mortality is approximately constant for each generation at all periods of life. Furthermore, in successive generations this constant relative mortality, characteristic of each, has tended to become lower with succeeding years of birth, until for people born in 1915 it was only 30 per cent of that of the standard period. We are thus led to a picture of the course of events which is somewhat unexpected in its general outline. It would seem that the actual calendar year is of relatively little importance in determining the improvement in the specific death-rates. What is of importance is the year of birth of the generation or group of individuals under consideration. Each generation after the age of 5 years seems to carry along with it the same relative mortality throughout adult life, and even into extreme old age.

It will be seen from Table 3 that these results are equally true of Scotland. In this case there is a slight complication due to the fact that in the earlier years there is a small increase of relative mortality in some of the higher ages, but the general tendency of the contours to run approximately along the diagonals is quite unmistakable.

If it be allowed that each generation is characterized by its own relative mortality-rate it becomes of interest to calculate values for this rate corresponding to different years of birth. We have done this by taking the average along each diagonal set of figures and in Figures 1 and 2 the resulting values have been plotted (Curve A) against years of birth for England and Wales and for Scotland respectively. It will be seen that for England and Wales the curve runs approximately constant at a value of about 100 per cent until the year 1835, when it begins to fall and continues to drop until for the group born in 1915 it reaches the value of 28 per cent. In the case of Scotland the exact point at which the curve begins to fall is slightly obscured by the fact that a slight rise seems to have occurred with its maximum in 1830. But the fall is well established by 1860, and the rate has dropped to 30 per cent in 1920. There is a suggestion that the fall in Scotland was about a decade behind that in England, but to some extent this result is dependent upon the somewhat arbitrary choice of the particular year which is taken

Table 4 Sweden: relative mortalities

AGE	1755				1805										1905			
	'65	'75	'85	'95	'15	'25	'35	'45	'55	'65	'75	'85	'95	'15	'25			
0	100	106	99	98	96	98	90	82	82	75	72	68	64	54	50	42	34	28
10	100	104	129	114	82	102	81	64	64	64	87	71	67	62	50	38	34	19
20	100	102	122	116	87	118	101	85	85	77	85	75	75	72	76	74	85	55
30	100	103	115	112	86	105	94	90	94	77	80	68	70	62	62	58	67	41
40	100	101	111	107	91	108	104	99	104	88	87	73	67	59	57	52	52	37
50	100	104	112	105	96	118	109	105	109	94	93	79	68	60	56	53	51	43
60	100	101	110	106	104	127	116	109	110	98	99	89	73	65	60	56	55	50
70	100	107	123	122	118	127	123	113	117	110	107	97	84	74	71	65	65	61

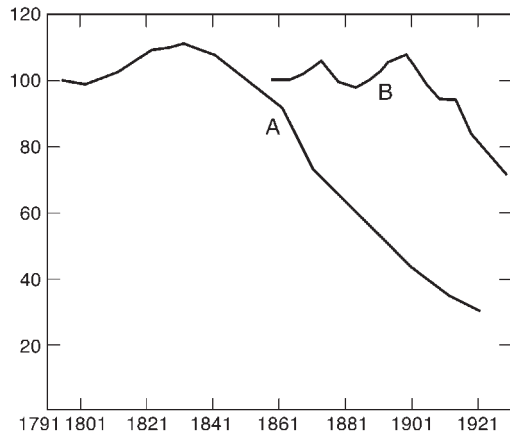


Figure 1 England and Wales: mean relative mortalities for various years of birth (Curve A) and relative infantile mortalities (Curve B)

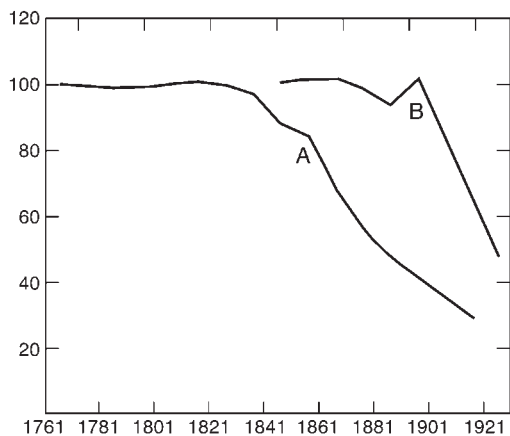


Figure 2 Scotland: mean relative mortalities for various years of birth (Curve A) and relative infantile mortalities (Curve B)

as standard, and so undue emphasis must not be laid upon it. It will be noted that so far no reference has been made to any ages below 5 years. Neither the infant death-rate nor the death-rates for the ages 1–4, suitably centred, fit into the above scheme. To elucidate the point we have plotted in the Figures (Curve B) the infantile death-rates, that is to say, deaths under 1 year per 1000 live births, for various periods. Both in England and Wales and in Scotland these death-rates remain approximately constant until about 1900, being approximately 150 in the former and 120 in the latter case. The rates shown in the Figures are relative rates, being calculated as percentages of the standard rates—137 and 119.6 per 1000 in England and Wales and in Scotland respectively. The fall since 1900 has been, both absolutely and relatively, greater in England than in Scotland. Clearly if the generalization suggested above included the infantile death-rates the two curves in each diagram would have been approximately identical. The fall in infantile death-rate was delayed relatively to the expected by more than a generation. In the case of the 1–4 age-group it is found that the fall in its death-rate was also somewhat delayed as compared with that expected, but to

a lesser extent. This group would seem to be intermediate in behaviour between the infants on the one hand and the older children and the adults on the other.

Interpretation of the regularities

We may now consider possible explanations of the regularities which have been described. Two broadly distinct possibilities suggest themselves, namely:

(1) That the consecutive improvements which have taken place in succeeding age-groups are the result of a series of independent sets of conditions, or legislative acts, and that the apparent regularity is largely fortuitous. It might for instance be suggested that early industrial legislation was directed towards the welfare of children, and that at a later date general industrial and social conditions improved, and that older people were last in being affected by industrial and housing changes. It may also be pointed out that some of the virulent infective diseases, such as small-pox and enteric fever, which have both practically disappeared, exerted a heavy toll on the children, and the removal of these diseases naturally exerted the greatest effect on that age-group; whilst the eradication of any disease conferring complete or almost complete immunity would affect, to an appreciable extent, only the younger age-groups. Although the summation of causes such as the above might produce an effect most marked in children, and a smaller effect in the adult population, nevertheless it would be somewhat surprising if the quantitative regularity just pointed out should emerge. The question therefore suggests itself as to whether the regularity has not its source in some more general feature of the population mechanism.

(2) It is clear that the regularities in the relative mortality-rates described in detail above imply a very remarkable statistical relationship. The figures behave as if the expectation of life was determined by the conditions which existed during the child's earlier years. For instance, we may postulate that, constant hereditary endowment being assumed, the health of the child is determined by the environmental conditions existing during the years 0–15, and that the health of the man is determined preponderately by the physical constitution which the child has built up. (This remark of course would hold on the assumption that the inherited vitality or constitution was remaining statistically constant. The question of the effects on the population of selection and other evolutionary processes which would become important when a longer period of time is taken into account are here neglected.) We suggest that this general hypothesis of improved child environment does give an adequate explanation of the observed figures, and we find it difficult to think of an equally satisfactory substitute.

An alternative assumption of a general character might be that a more healthy race of children was born in each successive decade, but apart from the inherent improbability of any substantial improvement in this respect taking place, at least before the development of prenatal welfare work in the present century, the suggestion would seem to be conclusively refuted by the fact that the improvement in the death-rates of children under one year has been considerably less in amount, and much longer delayed in its appearance than the improvements in children of upwards of one year. If the suggestion were true it would be expected that the increased vitality would show itself at least as

markedly during the first year as in any subsequent age-period, which it does not.

The anomalous behaviour of this first age-group is, however, consistent with the previous postulate of improved child environment. It has been observed above that the improvement in the infantile death-rate became apparent only after 1901. If we remember that before birth and during its first year of life, the child is dependent of its welfare to a very large degree upon the general health and vitality of the mother, then it would be expected that a substantial improvement in the health of the latter would show itself in a reflected improvement in the infantile death-rate. The mothers of 1901 would on the average be born about 1870 or possibly a little later, and, as mentioned above, the health of females born at that date had so far improved that the death-rate was reduced by about 30 per cent in the case of Scottish and by about 40 per cent in the case of English mothers. It is suggested that this may constitute at least one of the factors conducive to the improvement of infantile mortality in the present century.

It will thus be seen that all the figures are consistent with the hypothesis that the death-rates of the adolescent and adult depend on the constitution acquired during the first 15 years or so of life, and that this constitution has undergone a very substantial improvement, presumably as the result of the general raising of the standard of life, and the amelioration of social conditions. If the above hypothesis be correct it would be implied that the decreased death-rates were in fact largely the result of the improved physique of the population.

Prediction of future mortalities

A very interesting question and one of considerable practical importance now arises. It is clear that if the above generalization can be extrapolated into the future, then predictions of the decreases of the death-rates at the various ages are possible. In other words, if we assume that the diagonal law is essentially true it becomes an easy matter to predict death-rates for the various age-groups in the future. We simply determine the relative mortality-rate characteristic of a particular diagonal by averaging the rates already available along that diagonal. The specific mortality-rate for a particular age-group at a future time is then found by multiplying the specific death-rate for that age-group in the standard year by the appropriate factor. It is obvious of course that as we go into the future the specific rates for the lower ages cannot be determined by this method. To demonstrate the feasibility of the method we have calculated the specific death-rates for the 1931 period for Scotland from the information available up to and including the 1921 period. The observed and calculated figures are given below (Table 5).

By a similar method we have calculated the specific death-rates for 1941 and 1951 on the basis of the information available up to date (Table 6).

These figures are put forward not so much as definite predictions of the course of events but rather as a simple statement of what will happen if the diagonal lines continue to run as they have been doing during the past 40 or 50 years. It is clearly impossible to assert that this will happen, but if the hypothesis of causation outlined above be accepted as substantially correct, then the expectation that it will happen would seem to acquire a higher degree of justification. As in the case of most empirical

Table 5 Death-rates for 1931, as calculated and as observed

Both sexes, aged –	Death-rates compared with 1860–62 (= 100)		Death-rates per 1000	
	Observed	Calculated	Observed	Calculated
15–	34.5	35.5	2.8	2.9
25–	39.5	43.5	3.9	4.3
35–	51.0	54.5	6.1	6.4
45–	66.0	61.9	10.4	9.8
55–	81.0	76.5	22.0	20.8
65–75	88.5	91.9	53.6	55.7

Table 6 Calculated death-rates for 1941 and 1951 (per 1000)

Age-periods	1941	1951
15–	2.5	–
25–	3.5	3.0
35–	5.0	4.2
45–	8.4	6.7
55–	17.1	14.4
65–	43.8	38.0

statistical results, it is assumed that no abnormal agency of great magnitude suddenly appears. It will be noted that in the relative mortalities for 1910–1920 in the English figures certain systematic divergences occur in the mortalities relating to the 5–15, 15–25, and 25–35 age-groups. Detailed examination of the statistics shows that this is accounted for by the abnormal figures of the war years. The Scottish figures do not show this effect because they refer to the periods 1910–12 and 1920–22, and so miss out the war years. On the whole it is surprising, in view of the great changes that have occurred during the period covered by the data, that there is not more evidence of substantial disturbing influences.

Relative mortalities in Sweden

In the case of Sweden the death-rates are available from the year 1750, and so it is natural to turn to the statistics of this country in order to find whether the above generalization holds for a country other than Great Britain. In Table 4 are shown the relative mortality-rates for Sweden calculated as in Tables 2 and 3. At first sight it would seem that the picture is quite a different one, but on closer examination it will be found that there is a disturbance limited to a rectangular block dating from 1855 onwards, and affecting the age-groups centred at 10, 20, and 30 years. When this block is omitted it will be found that the contours run with moderate regularity down the diagonals. The first contour below 100 begins in 1815—that is, refers to persons born in 1805—so that it would seem that the improvement in health began in Sweden at an earlier date than in Great Britain. During the half of the century previous to 1800, the figures indicate an essentially static state, with fluctuations in which all age-groups tend to be affected proportionally at the same time. We are not sure to what extent those ‘vertical’ changes are real, or to what extent they may have been due to statistical errors, such as wrong estimates of the population. If we assume that the abnormal figures from 1855 onwards are due to some adventitious circumstance at present unknown, we may proceed by omitting this rectangle and use the remaining

figures in order to calculate the relative mortality-rates characteristic of each year of birth as we have done above for Great Britain. The result is shown in Figure 3 (Curve A), which may be compared with Figures 1 and 2.

As in the former cases, we have also shown the infantile mortality-rates (Curve B). In the case of Sweden both mortality-rates began to fall much earlier than in England and Wales and in Scotland, and again the fall in the infantile mortality would seem somewhat to post-date that in the relative mortality-rate, though possibly not in so striking a fashion as in the other countries. The general fall in the relative mortality in Sweden has been progressive but less steep than in the other two countries.

Reference may be made at this point to a paper by Greenwood⁴ in which he describes and contrasts the vital statistics of Sweden and of England and Wales from 1850–1910. He concentrates attention on the two age-groups centred at 20 and 50, and shows that whereas in the former Sweden started by being better than England and finished by being considerable worse, in the latter the positions were reversed, and England which began superior was in the end definitely inferior to Sweden. If the available statistics are viewed from the more comprehensive point of view suggested in the present paper the same facts take on a somewhat different complexion. The contrast in behaviour since 1850 is seen to depend upon the earlier beginning in Sweden of a less rapid decline in death-rate, combined with the peculiar disturbance exhibited by the lower age-groups from 1855 onwards. Although this paper does not set out to deal primarily with the vital statistics of Sweden, it is suggested that investigation of these from the present point of view may shed light on the causes of the changes which have been going on in that country, and help to focus attention on the really abnormal features which its vital statistics exhibit.

Discussion

Possibly the most arresting result of the predictions embodied in Table 6 is the suggested substantial decline in the death-rate of that section of the population above 65 years of age. Many may consider that such a reduction is most unlikely and entirely out of harmony with experience up to date. It seems in some quarters to be accepted as a fact that although very substantial improvements have been effected in the death-rates for the age-groups up to 65 or 70, yet for ages above 70 there are no indications that any improvement is likely. That this view is

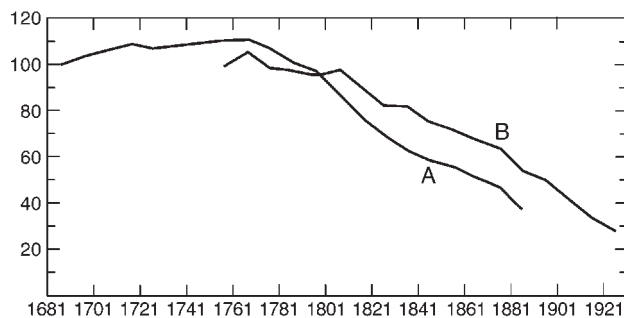


Figure 3 Sweden: mean relative mortalities for various years of birth (Curve A) and relative infantile mortalities (Curve B)

not substantiated by the facts is shown by a consideration of Tables 2 to 4 in Scotland (Table 2) the improvement so far recorded is scarcely significant, the relative mortality in the group aged 65–75 having in 1930 fallen to 88.5 per cent, but as the individuals under consideration were born about 1861 a marked improvement would scarcely be expected. In England and Wales (Table 3) the improvement is more definite though still rather small in degree. In Sweden (Table 4) however, we have for this group a very much more substantial reduction in the relative mortality-rate (a fall to 61 per cent). The fact that this fall is so much more marked in Sweden corresponds with the much earlier onset of the process of improvement in that country. It is clear therefore that there is no inherent impossibility in the suggestion that the death-rates for these higher age-groups are likely to undergo very substantial reductions during the next few decades. It is generally recognized that with the present distribution of birth-rates and death-rates there will be a steady increase in the percentage of old people in the community, and it is obvious that a decrease in the death-rates in the higher ages, such as these predictions foreshadow, will very markedly accentuate this tendency.

The thorough investigation of the many problems raised in the course of this work will necessarily entail considerable time and labour. Two directions suggest themselves as being most likely to be of use in determining what is the real explanation of these phenomena—first the investigation of the statistics of a larger number of different countries from the same point of view, and secondly the detailed examination of the causes of death at different periods of different age-groups in any one country. Analysis of the Scottish records from the latter point of view is proceeding. It appears desirable that this should be taken up in the case of any other country for which records are available.

It might seem at first sight that our choice of the first period for which statistics were available as the standard period was somewhat arbitrary. It implies for instance that the process of improvement had not yet begun, and though this is fairly obviously true in the case of Sweden and possibly of England and Wales it is more doubtful in the case of Scotland. In the light of our general conclusions it might seem more justifiable to choose as the standard rates not those existing in any one year, but a series obtained by following a diagonal. The resulting series of standard rates so found would involve no assumption as to the steadiness of the conditions existing, except that influences occurring from year to year, such as would seem to exist in the earlier periods in Sweden, would cause greater random interference. This difficulty, however, could be got over by taking all possible diagonals and finding the best fitting set of standard figures by suitable methods of calculation. In this paper we have not thought it necessary or even desirable to adopt this procedure. Had we done so, it might have appeared to the reader that the resulting diagonal lines were artefacts depending on the circumstance that diagonal arrays were chosen as the starting point. The simple and direct method which we have followed seems to avoid this difficulty. Furthermore, we find that the use of diagonal arrays from the beginning does not in fact substantially modify the final results.

It is our purpose here to emphasise the medical rather than the actuarial interest of the regularities we have been describing, but it is obvious that these results have a bearing on

actuarial problems, and it is not surprising that results, related to these, have been arrived at in this connexion, though with a somewhat different method of approach. Reference may here be made to a paper by one of us (McKendrick⁵) in which, from a general schematic representation of the process of population change, and its mathematical treatment, it becomes clear that expectation of life, etc. should be determined from a series of mortality-rates appearing down the diagonals, and not down the columns. In a paper by AR Davidson and AR Reid⁶ emphasis is laid on the necessity of considering the death-rates characterizing a generation, that is, a group of people born in a particular year, rather than the mortalities of a particular calendar year, and in the discussion on this paper Mr AE King put forward hypothetically as possibly approximating to the facts, a suggestion which is essentially equivalent to our result. In a paper by VPA Derrick,⁷ emphasis is again laid on the advantage of considering generation mortalities and not ordinary mortalities. He shows that much more nearly similar curves are obtained when the variable employed is the year of birth and not the calendar year, and so, approaching the problem from a somewhat different angle from our own, he has arrived at what is substantially the same result. Nevertheless the method of presentation developed in the present paper seems to demonstrate the phenomenon in a more direct and striking manner than any which we have found in the literature, and further, it seems important that it ought to be brought to the notice of those who are concerned with the health services of this country, whatever be the interpretation which may be given, or the value assigned to it in connexion with the prediction of future events.

From the formal point of view the result is a very simple one. It appears that the specific death-rate is the product of two factors, one of which is a function of the age alone, and the other a function of the year of birth alone.

Practical inferences

The facts represented in Tables 2 and 3 with regard to England and Wales and to Scotland are quite indisputable, but we would emphasize that the interpretation given above is in the nature of a hypothesis which fits the facts. If this hypothesis be accepted as substantially correct, it is interesting to note that certain conclusions of great importance from the medical and public health point of view would seem to follow. It would be implied that care of the children during their first 10–15 years of life is of supreme importance. It is at this period of life that improved environment exercises its effect most promptly, and furthermore the improved physique built up during this period would seem to be of decisive effect at all later ages.

At the same time it would be a mistake in the present state of knowledge to push this argument too far and to deprecate care of the adult and to spend all our effort on the care of the child. It must be remembered that the statistics which have been presented are compatible with the existence of secondary factors which may wholly or partly neutralize one another. For example, it seems reasonable to believe that in a favourable environment the standard of health for survival through the younger age-periods would be lower than in a less favourable one. Thus one would expect that a larger number of weaker individuals would go on to the higher age-groups. Nevertheless there is no

indication in the statistics that the death-rates in the higher age-groups are adversely affected by the presence therein of those weaker individuals. Possible the persistence of the favourable environment may counterbalance this effect and facilitate the survival of those weaker individuals so that their presence is not statistically noticeable. Of course the process suggested here is not to be confused with the general hypothesis described above—namely, that good environment in childhood builds up a stronger constitution and raises the standard of physique of the adolescent to a substantial degree. The possibility raised in this paragraph emphasizes the importance of maintaining in any modern community good conditions for the adult as well as for the child.

Conclusions

(1) In England and Wales and in Scotland the improvement attained at any particular time in the death-rates of the various age-groups depends primarily upon the date of birth of the individuals concerned, and only indirectly upon the particular year under consideration. An exception occurs in the case of the infantile death-rates, which show definite lags in their dates of improvement.

(2) It is shown that these results are consistent with the hypothesis that the important factor from the point of view of the health of the individual during his whole life is his environment up to the age of say 15 years, and that improved conditions at later ages have little direct effect. Improved conditions appear to have brought about their beneficial results primarily through their action on the children.

(3) It is suggested that improvement in infantile mortality is dependent in large measure on improvement in maternal health. This, apart from other influences, would explain why the infantile mortalities lagged behind in their fall.

(4) The figures for Sweden do not show the same simple regularities, but if a section of the table be omitted (relative mortalities for the age-groups centred at 10, 20, and 30 years from 1855 onwards), the rest of the table exhibits the characteristic features described under (1). It would appear that in the case of Sweden a disturbing force began to operate about 1850 and adversely affected the adolescents and young adults.

(5) The facts described under (1), and the hypothesis suggested in (2) direct attention to the importance of good environmental conditions during childhood.

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